THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Time: 3 Hours **Wednesday 12 May 2004 p.m.**

Instructions

1. This paper consists of sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All necessary steps in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae, slide rules and nonprogrammable pocket calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet(s).

This paper consists of 4 printed pages.

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SECTION A (60 marks)

Answer all questions from this section showing all necessary steps and answers.

1. (a) Use logarithms to evaluate to three significant figures:

$$\left(\frac{\ln 32 - \sqrt[3]{0.06e^2}}{\tan 55^\circ 36'}\right)^{0.5}$$

(Hint: $\ln x = \log x \div \log e$)

(04

(b) Using a non programmable scientific calculator, find the value of $\sqrt{\frac{e^5 \sqrt{\ln 32}\log 32}{\sqrt{3}}}$ correct to 5 decimal places.

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2. (a) Using letters and logical connectives, write the following statement. "If x is less than zero then it is not positive."

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(b) Using the statement given in question 2.(a), find and simplify:

(i) the contrapositive of its inverse.

(ii) the converse of its contrapositive.

(iii) comment on the resulting statements in 2.(b)(i) and 2.(b) (ii) above.

(04 mark

3. (a) Find the centre and radius of the circle given by the equation $x^2 + y^2 + 4x - 8y + 4 = 0$.

(03 marks)

(b) Find the length of the tangent from the point (3,8) to the circle whose equation is given in question 3.(a).

(03 marks)

4. (a) Prove that

$$\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$$

(02 marks)

(b) Solve for x if $\tan(\cos^{-1}x) = \sin(\tan^{-1}2)$

(04 marks)

5. (a) Using synthetic division find the value of c given that the polynomial $p(x) = x^3 + cx^2 - 2cx + 4$ is divisible by $x - 1$.

(02 marks)

(b) The expression $x^3 + ax^2 + bx + c$ gives the same remainder when divided by $x + 1$ or $x - 2$. Show that

(i) $a + b = -3$

(ii) Find c if the expression leaves the remainder of 7 when divided by $x - 1$. (04 marks)

6. (a) Show that the equation $3y^2 - 10x - 12y = 18$ represents a parabola.

(03 marks)

(b) Find the equation of the tangent through the point $\left(\frac{3}{\sqrt{2}}, 2\right)$ on the ellipse $8x^2 + 9y^2 = 72$.

(03 marks)

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7. (a) Find the values of z for which $12\cosh^2 z + 7\sinh z = 24$ **(04 marks)**

(b) If $y = A\cosh nx + B\sinh nx$, prove that $\frac{d^2y}{dx^2} = n^2y$. **(02 marks)**

8. The table below represents the height taken to the nearest centimeter of 40 orange trees in a garden.

Height (cm)	131 – 140	141 – 150	151 – 160	161 – 170	171 – 180	181 – 190	191 – 200
Number of trees	3	4	7	11	9	5	1

(a) Using the assumed mean A, calculate the actual mean height.

(b) Calculate the Standard deviation of the distribution. **(06 marks)**

9. A factory finds that on average 20% of the bolts produced by a given machine will be defective for certain specified requirements. If 10 bolts are selected at random from the day's production of this machine, find the probability that:

(i) 2 or more will be defective.

(ii) More than 5 will be defective. **(06 marks)**

10. (a) Express $\sin 5\theta$ and $\cos 5\theta$ in terms of $\sin\theta$ and $\cos\theta$ and hence show that:

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 + 5\tan^4\theta - 10\tan^2\theta}$$ **(05 marks)**

(b) Write the complex number $z = x + iy$ in polar form. **(01 mark)**

SECTION B (40 marks)

Answer **four (4)** questions from this section showing all necessary steps and answers.

11. (a) Given the equation of a line as $\frac{x+1}{4} = \frac{y-2}{-1} = \frac{z+0}{5}$, find the equation of the plane that contains the point (½, 0, 3) and is perpendicular to the line which is both parallel to the vector $2\underline{i} - \underline{j} + 3\underline{k}$ and passes through the point (5, –2, 4). **(04 marks)**

(b) (i) The position vectors of points P and Q are $3\underline{i} + \underline{j} + 2\underline{k}$ and $\underline{i} - 2\underline{j} - 4\underline{k}$ respectively. Find the equation of the plane through B and perpendicular to $\overline{AB}$. **(04 marks)**

(ii) Find the vector equation of a line through the point A with position vector $\underline{a} = 3\underline{i} - 2\underline{j} + 3\underline{k}$ and parallel to the vector $\underline{b} = 4\underline{i} + \underline{j} - 2\underline{k}$. **(02 marks)**

12. (a) Find the inverse of the matrix A if $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & -2 \\ 3 & 1 & -1 \end{pmatrix}$ **(05 marks)**

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$(i)^2 \times (i)^2$

(b) Use the inverse obtained in question 12.(a) above to solve the system of the following equations.

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

1 − 4 + 9 = 6 ✓
2 + 6 + 6 = 14 ✓
3 − 2 − 3 = −2 ✓

(05 marks)

13. (a) Find the equation of the chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ joining the points $(a \sec\theta, b\tan\theta)$ and $(a\sec\alpha, b\tan\alpha)$. Hence deduce the equation of a tangent at the point $(a\sec\theta, b\tan\theta)$.
(05 marks)

(b) Show that the line $3x - 4y = 5$ is a tangent to the hyperbola $x^2 - 4y^2 = 5$ and find the point of contact.
(02 marks)

14. (a) Using the definitions of cosh x and sinh x, show that:

(i) $\cosh^2 x - \sinh^2 x = 1$.

(ii) $\cosh^{-1} x = \pm \log\left(x + \sqrt{x^2 - 1}\right)$.

(05 marks)

(b) Calculate the minimum value of the function
$y = 3\cosh x + 2\sinh x$.
(05 marks)

15. (a) Simplify the following using the laws of algebra of propositions.

(i) $P \vee (P \wedge q)$

(ii) $\sim(P \vee q) \vee (\sim P \wedge q)$

(08 marks)

(b) Translate the following argument into symbolic form. Hence show that the argument is valid.
(02 marks)

"On my daughter's birthday, I bring her flowers. Either it is my daughter's birthday or I work late. I did not bring my daughter flowers today. Therefore, today I worked late".

16. (a) (i) Find:
$$\int \frac{xe^x}{(1+x)^2}\,dx.$$
(03 marks)

(ii) Evaluate $\int_{\frac{1}{2}}^{3} x\sqrt{2x+3}\,dx$.
(03 marks)

(b) The finite region bounded by the y – axis, the line $y = 27$ and the curve $y = \frac{1}{8}x^3$ is rotated completely about the y axis. Find the volume swept out.
(04 marks)

$y^3 - 216 = 0$

$3/2 + 1 = 5/2$

9

4

x . (. :

(I) CHAMBO

$1 - \frac{1}{4} = \frac{4-1}{4}$ ✓